Mathematical Science

**MATH 5301** Elementary Analysis I (3 semester credit hours) Sets, real numbers, metric spaces, topology of Euclidean space, continuity and differentiability of functions of a single variable, uniform convergence, sequence and series of functions. Prerequisite: One year of calculus through multivariable calculus or instructor consent required. (3-0) Y

**MATH 5302** Elementary Analysis II (3 semester credit hours) Riemann and Darboux integrals, functions of bounded variation, Riemann-Stieltjes integration, Lebesgue measure, Introduction to Lebesgue integral. Prerequisite: **MATH 5301** or **MATH 4301**. (3-0) Y

**MATH 5303** Advanced Calculus and Linear Algebra (3 semester credit hours) Concise introduction to elementary functions; differentiation; simple integration techniques; improper integrals; series and sequences; convex functions. Systems of linear equations, eigenvectors, and spectral theorem for normal matrices. Partial derivatives and linear approximations; optimization in one or several variables; multiple integrals. Applications of calculus and matrix algebra to differential equations and geometry of curves and surfaces. Prerequisite: At least one semester of undergraduate calculus or instructor consent required. (3-0) Y

**MATH 5304** Applied Mathematical Analysis for Non-Majors (3 semester credit hours) Techniques of mathematical analysis applicable to the social, behavioral and management sciences. Differential and integral calculus of one and many variables. May not be used to fulfill degree requirements. Prerequisite: College algebra or instructor consent required. (3-1) S

**MATH 5305** Practical Applications in Higher Geometry (3 semester credit hours) Topics in modern Euclidean geometry including distinguished points of a triangle, circles including the nine-point circle, cross ratio, transformations; introduction to projective geometry. May not be used to fulfill degree requirements for mathematical sciences majors except those in the Master of Arts in Teaching (MAT) program. Prerequisite: Junior-level mathematics course. (3-0) T

**MATH 5306** Practical Applications in Non-Euclidean Geometry (3 semester credit hours) The relations among elliptic, Euclidean and hyperbolic geometries, Euclidean models of elliptic and hyperbolic geometries. May not be used to fulfill degree requirements for mathematical sciences majors except those in the Master of Arts in Teaching (MAT) program. Prerequisite: Junior-level mathematics course. (3-0) T

**MATH 5313** Modern Algebra for Teachers (3 semester credit hours) Study of modern algebra involving groups, rings, fields and Galois Theory. May not be used to fulfill degree requirements for mathematical sciences majors except those in the Master of Arts in Teaching (MAT) program. Prerequisite: Junior-level mathematics course. (3-0) R

**MATH 5390** Topics in Mathematics - Level 5 (3 semester credit hours) May be repeated for credit as topics vary. Instructor consent required. (3-0) R

**MATH 6301** Real Analysis (3 semester credit hours) Lebesgue measure in finite-dimensional spaces, Abstract measures, measurable functions, convergence a.e., Egorov's Theorem, convergence in measure, Lebesgue integral, Lebesgue's bounded convergence theorem, Levi’s monotone convergence theorem, Fatou's Lemma, Fubini's theorem, Lp-spaces. Prerequisite: **MATH 5302**. (3-0) Y
MATH 6302 Functional Analysis I (3 semester credit hours) Banach and Hilbert spaces, classical theorems of functional analysis, compact operators, Fredholm operators, elements of spectral theory, introduction to unbounded operators. Prerequisite: MATH 6301. (3-0) Y

MATH 6303 Theory of Complex Functions I (3 semester credit hours) Complex integration, Cauchy's theorem, calculus of residues, power series, entire functions, Riemann mapping theorems. Riemann surfaces, conformal mapping with applications. Prerequisites: MATH 5301 and MATH 5302 or instructor consent required. (3-0) Y

MATH 6304 Theory of Complex Functions II (3 semester credit hours) Riemann surfaces, meromorphic and holomorphic functions and differentials, the normalization theorem, the Riemann-Roch theorem, Abel theorem, applications to nonlinear equations. Prerequisite: MATH 6303. (3-0) T

MATH 6305 Mathematics of Signal Processing (3 semester credit hours) The course is devoted to a mathematical foundation of some of the key topics in signal processing: discrete and continuous signal transforms, least squares methods and adaptive filtering, compressed sensing and related topics. Prerequisites: Linear algebra and calculus through multivariate calculus or instructor consent required. (3-0) T

MATH 6307 Wavelets and Their Applications (3 semester credit hours) An introduction to windowed Fourier and continuous wavelet transforms, generalized frames, discrete wavelet frames, multiresolution analysis, Daubechies' orthogonal wavelet bases, and their applications in partial differential equations and signal processing. Prerequisites: Two semesters of calculus and differential equations or instructor consent required. (3-0) T

MATH 6308 Inverse Problems and Applications (3 semester credit hours) Exact and approximate methods of nondestructive inference, such as tomography and inverse scattering theory in one and several dimensions, with applications in physical and biomedical sciences and engineering. Prerequisites: Two semesters of calculus and differential equations or instructor consent required. (3-0) T

MATH 6309 Differential Geometry (3 semester credit hours) Smooth manifolds, tangent bundles, smooth partitions of unity, submanifolds, Sard's theorem, transversality, embeddings, Whitney theorem, differential forms, Frobenius Theorem, de Rham cohomology, degree theory on manifolds, Riemannian metric, Gauss-Bonnet theorem. Prerequisite: MATH 5301 or instructor consent required. (3-0) T

MATH 6310 Topology (3 semester credit hours) Metric spaces, introduction to topology, elements of homotopy theory, covering spaces, fundamental group, homotopy groups, fibrations, simplicial complexes and CW-complexes, degree theory. Prerequisite: MATH 5301 or instructor consent required. (3-0) Y

MATH 6311 Abstract Algebra I (3 semester credit hours) Basic properties of groups, rings, fields, and modules. Prerequisite: Two semesters of undergraduate abstract algebra or instructor consent required. (3-0) Y

MATH 6312 Combinatorics and Graph Theory (3 semester credit hours) This course covers theory and applications of combinatorics and graphs, topics from basic counting principles, principle of inclusion and exclusion, permutation statistics, ordinary and exponential generating functions, composition of integers, integer partitions, Stirling numbers of the first kind, q-analogs of binomial and multinomial coefficients, Euler's formula, Hamilton paths, planar graphs, chromatic and Tutte polynomials and algorithms on networks. Prerequisites: Theoretical Concepts of Calculus and Abstract Algebra I is required or instructor consent required. (3-0) T
**MATH 6313** Numerical Analysis (3 semester credit hours) A study of numerical methods including the numerical solution of non-linear equations, interpolation, approximation by polynomials, numerical integration. Numerical solution of ordinary differential equations including initial value problems and two-point boundary value problems. Prerequisites: Knowledge of a high-level programming language and linear algebra and calculus through multivariable calculus and department consent required. (3-0) Y

**MATH 6314** Algebraic Topology (3 semester credit hours) This course covers basics in algebraic topology. Topics will include simplicial and singular homology groups, cellular homology groups, exact sequences and excision, chain maps, Mayer-Vietoris sequences, homology with coefficients, Eilenberg-Steenrod axioms; cohomology theory, the universal coefficient theorem, cup products, Kunneth formulas, and Poincare duality. Prerequisites: Abstract Algebra I and Topology or equivalent is required or instructor consent required. (3-0) T

**MATH 6315** Ordinary Differential Equations (3 semester credit hours) The study of ordinary differential equations with emphasis on existence, uniqueness, linear systems, boundary value problems, and stability. Prerequisites: Linear algebra and differential equations and MATH 5302 or instructor consent required. (3-0) Y

**MATH 6316** Differential Equations (3 semester credit hours) Continuation of MATH 6315 and an introduction to partial differential equations. Prerequisite: MATH 6315. (3-0) T

**MATH 6318** Numerical Analysis of Differential Equations (3 semester credit hours) Practical and theoretical aspects of numerical methods for partial differential equations are discussed. Topics selected from: finite difference, finite element and boundary element approximations for partial differential equations. Application of methods will be illustrated using MATLAB. Prerequisite: MATH 6313 or equivalent. (3-0) T

**MATH 6319** Principles and Techniques in Applied Mathematics I (3 semester credit hours) Mathematical methods usually used in applied sciences and engineering. Topics chosen from advanced linear algebra; Hilbert spaces; positivity; quaternions; integral equations; Fourier analysis; distributions; convexity; asymptotic methods; special functions. Prerequisites: Linear algebra and differential equations or instructor consent required. (3-0) T

**MATH 6320** Principles and Techniques in Applied Mathematics II (3 semester credit hours) Continuation of Math 6319. Prerequisite: MATH 6319. (3-0) T

**MATH 6321** Optimization (3 semester credit hours) Introduction to theoretical and practical concepts of optimization in finite and infinite dimensional setting, least-squares estimation, optimization of functionals, local and global theory of constrained optimization, iterative methods. Prerequisites: Linear algebra or instructor consent required. (3-0) T

**MATH 6322** Mathematical Foundations of Data Science (3 semester credit hours) Graphs, topological polyhedra; homology of cubical sets; computations of homology groups, rational functions and interval arithmetic, maps on intervals, chain selectors; homology of maps; persistence diagram, applications to digital image processing, images and cubical sets, time-dependent patterns, and size function. Prerequisites: (MATH 6312 or equivalent) and instructor consent required. (3-0) Y

**MATH 6324** Applied Dynamical Systems I (3 semester credit hours) Topics from the theory of discrete time dynamical systems including symbolic dynamics, chaos, box counting dimension and fractals, bifurcations, period doubling route to chaos, Sharkovsky's theorem, Lyapunov exponents, maps of the circle and synchronization, area preserving maps, invariant curves, and strange attractors. Topics selected from the singularity theory and the theory of continuous time dynamical systems. Examples of models from ecology,
epidemiology, economics, and engineering are presented. Prerequisite: MATH 6301. (3-0) T

MATH 6325 Nonlinear Analysis I (3 semester credit hours) Topological degree in finite dimensions and applications to intermediate value theorem in dimension n > 1, Fundamental Theorem of Algebra, Argument Principle in Complex Analysis, Brouwer fixed point theorem, Poincare-Bendixson Theorem on periodic solutions to ODEs, Lyapunov stability of equilibrium, guiding function method, Leray-Schauder degree, solvability of boundary value problems, and bifurcation theory. Prerequisite: MATH 6301. (3-0) T

MATH 6327 Stability and Bifurcations of Switched Systems (3 semester credit hours) This course will cover finite-time, asymptotic, and global stability of equilibria of switched systems, switched equilibria, stability of limit cycles of switched systems (including stick-slip oscillations and cycles with jumps), dimension reduction, and extension to larger classes of nonlinear switched systems via bifurcation theory. Prerequisites: Differential Equations and Multivariable Calculus and instructor consent required. (3-0) T

MATH 6331 Mathematics of Signals, Systems, and Controls (3 semester credit hours) Basic principles of systems and control theory: state space representations, stability, observableness, controllability, realization theory, transfer functions, and feedback. Prerequisites: Linear algebra and differential equations or instructor consent required. (3-0) T

MATH 6332 Advanced Control (3 semester credit hours) Theoretical and practical aspects of modern control methodologies in state space and frequency domain, in particular LQG and H-infinity control: coprime factorizations, internal stability, Kalman filter, optimal regulator, robust control, sensitivity minimization, loop shaping, model reduction. Prerequisite: MATH 6331. (3-0) T

MATH 6336 Nonlinear Control Systems (3 semester credit hours) Differential geometric tools, input-output maps, feedback linearization, nonlinear observers, input-output linearization, output tracking, and regulation, passivity based control, control systems on Lie groups. Prerequisites: (MATH 6315 and MATH 6331) or instructor consent required. (3-0) T

MATH 6338 Delay Differential Equations (3 semester credit hours) Delay differential equations (DDEs) describe the phenomenon that the rate of change of the state variable is dependent on its historical memory. Course topics will be selected from: Existence and uniqueness of solutions, continuation and continuous dependence on parameters of solutions; Linear systems of delay differential equations; Basic notions of dynamical systems induced by DDEs; Periodic solutions and Hopf bifurcation; Analysis of DDE models; DDEs with state-dependent delays and their local and global Hopf bifurcation. Prerequisite: MATH 6315, or instructor's consent. (3-0) T

MATH 6339 Control of Distributed Parameter Systems (3 semester credit hours) Theoretical and technical issues for control of distributed parameter systems in the context of linear infinite dimensional dynamical systems. Evolution equations and control on Euclidean space, elements of functional analysis, semigroups of linear operators, abstract evolution equations, control of linear infinite dimensional dynamical systems, approximation techniques. Prerequisites: partial differential equations and MATH 5301 or instructor consent required. (3-0) T

MATH 6340 Numerical Linear Algebra (3 semester credit hours) Topics include direct and iterative methods for solving linear systems; vector and matrix norms; condition numbers; least squares problems; orthogonalization, singular value decomposition; computation of eigenvalues and eigenvectors; conjugate gradients; preconditioners for linear systems; computational cost of algorithms. Topics will be supplemented with programming assignments. Prerequisites: Knowledge of a high-level programming language and linear algebra and numerical analysis or instructor consent required. (3-0) Y
**MATH 6341** Bioinformatics (3 semester credit hours) Fundamental mathematical and algorithmic theory behind current bioinformatics techniques are covered and implemented. They include hidden Markov models, dynamic programming, genetic algorithms, simulated annealing, neural networks, cluster analysis, and information theory. Prerequisites: Knowledge of Unix and a high level programming language. (3-0) T

**MATH 6342** Scientific Computing (3 semester credit hours) Introduction to scientific computing through projects in computational science and engineering. Topics include mathematical modeling; theoretical analysis of such models; numerical and symbolic computation; verification and validation; computational simulation. Representative projects will include applications of dynamical systems, Monte Carlo simulations, numerical optimization, and linear and nonlinear partial differential equations. The course includes an introduction to symbolic computation and to programming in MATLAB, Python, and/or C. Some prior programming experience is recommended. Prerequisites: Prior courses in numerical analysis and partial differential equations and **MATH 6315** or instructor consent required. (3-0) T

**MATH 6343** (BMEN 6389 and BIOL 6385) Computational Biology (3 semester credit hours) Machine learning and probabilistic graphical models have become essential tools for analyzing and understanding complex systems biology data in biomedical research. This course introduces fundamental principles and methods behind the most important high throughput data analysis tools. Applications will cover molecular evolutionary models, DNA/protein motif discovery, gene prediction, high-throughput sequencing and microarray data analysis, computational modeling gene expression regulation, and biological pathway and network analysis. Prerequisite: Some background in elementary statistics/probability or introductory bioinformatics, or instructor consent required. (3-0) Y

**MATH 6345** Mathematical Methods in Medicine and Biology (3 semester credit hours) Introduction to the use of mathematical techniques in solving biologically important problems. Some examples of topics that might be covered are biochemical reactions, ion channels, cellular signaling mechanisms, kidney function, and nerve impulse propagation. Prerequisite: One year of calculus is required with differential equations recommended or instructor consent required. (3-0) T

**MATH 6346** Medical Image Analysis (3 semester credit hours) Introduction to mathematical and computational methods in extracting clinically useful information from medical images. Topics include image enhancement, feature extraction and shape analysis, image segmentation algorithms used to localize and identify target structures in medical images, image registration algorithms used to determine the correspondence of multiple images of the same anatomical structure, and image classification. Prerequisites: Linear algebra and calculus through multivariable calculus, or consent of the instructor. (3-0) Y

**MATH 6350** Quantum Computation and Information (3 semester credit hours) Quantum states, channels, measurements; entropy, subadditivity; entanglement measures, discord; teleportation, dense coding, quantum key distribution; Shor's algorithm, Grover's search algorithm, hidden subgroup algorithms. Prerequisite: Linear algebra or instructor consent required. (3-0) T

**MATH 6364** Stochastic Calculus in Finance (3 semester credit hours) Brownian Motion, Ito Calculus, Feynman-Kac formula and an outline of Stochastic Control, Black Scholes Analysis, Transaction Costs, Optimal Portfolio Investment. Prerequisite: **STAT 5351** or instructor consent required. (3-0) T

**MATH 6390** Topics in Mathematics - Level 6 (3 semester credit hours) May be repeated for credit as topics vary. Instructor consent required. (3-0) R

**MATH 6V81** Special Topics in Mathematics - Level 6 (1-9 semester credit hours) May be repeated for credit as
topics vary. Instructor consent required. ([1-9]-0) S

**MATH 6V98** Masters Thesis (3-9 semester credit hours) Pass/Fail only. May be repeated for credit. Instructor consent required. ([3-9]-0) S

**MATH 7309** Knot Theory (3 semester credit hours) This course covers basics in combinatorial knot theory. Topics will include: ambient isotopy, basic invariants of knots and links in S3, polynomial invariants of knots and links, fundamental group of link complement, fundamental group of cyclic branched covers, Fox calculus, Alexander matrix, and categorification of polynomial invariants and their properties. Prerequisites: **MATH 6310** and **MATH 6311** or instructor consent required. (3-0) T

**MATH 7313** Partial Differential Equations I (3 semester credit hours) Classical and modern solution techniques for initial and boundary value problems for parabolic, elliptic, and hyperbolic linear partial differential equations. Existence, uniqueness, well-posedness, fundamental solutions, and Green's functions. First-order nonlinear equations, scalar conservation laws, and the method of characteristics. An introduction to weak solutions and the theory of Sobolev spaces. Prerequisite: **MATH 6301** and Math 6315 or equivalent. (3-0) T

**MATH 7314** Partial Differential Equations II (3 semester credit hours) Continuation of **MATH 7313**. Prerequisite: **MATH 7313**. (3-0) T

**MATH 7316** Wave Propagation with Applications (3 semester credit hours) Study of the wave equation in one, two and three dimensions, the Helmholtz equation, associated Green's functions, asymptotic techniques for solving the propagation problems with applications in physical and biomedical sciences and engineering. Prerequisites: **MATH 6303** and **MATH 6318** or equivalent. (3-0) T

**MATH 7318** (OPRE 7318) Stochastic Dynamic Programming (3 semester credit hours) Stochastic Dynamic Programming (SDP) is a general methodology which plays an essential role in many areas of economics and management science. The course provides students with a solid background on SDP, the core theory and its evolution and applications. The course discusses many models, particularly in finance and operations management, as well as additional concepts such as principal-agent concepts for dynamic systems. Instructor consent required. (3-0) Y

**MATH 7319** Functional Analysis II (3 semester credit hours) Topological vector spaces, locally convex spaces, Frechet spaces, test function spaces and tempered distributions, Fourier transforms and applications to differential equations. Recommended Prerequisite: **MATH 6303**. Prerequisites: **MATH 6301** and **MATH 6302**. (3-0) T

**MATH 7325** Nonlinear Analysis II (3 semester credit hours) This course covers elements of the equivariant topology, Burnside ring and the related algebraic structures, Euler ring, equivariant degrees. This subject has applications to differential equations, symmetric Hopf bifurcation theory and critical point theory. Prerequisite: **MATH 6325**. (3-0) T

**MATH 7329** Topological and Algebraic Methods in Nonlinear Differential Equations (3 semester credit hours) This course covers Polynomial homogeneous systems of ODEs, Poincare index, elliptic, hyperbolic and parabolic sectors, Bendixson formula, classification of plane quadratic systems, Riccati equation in non-associative commutative algebras, nilpotents and equilibria, idempotents and ray solutions, complex structures in algebras and bounded/periodic regimes, applications to Kasner equation, Euler equation and second order chemical reactions. Prerequisite: **MATH 6315**. (3-0) T

**MATH 7361** Algebraic Geometry and Non-linear Equations (3 semester credit hours) This course covers
Theta-functions of one variable, Analytic construction of the Jacobian of a compact Riemann surface, Related theta-functions, Algebraic construction of the hyperelliptic Jacobians, C. Neumann dynamical system, Characterization of the hyperelliptic period matrices, Soliton equations, The Riemann-Schottky problem and the Novikov conjecture. This subject has applications to Mechanics, Geometry, and Cryptography. Prerequisite: MATH 5301 or MATH 6301. (3-0) T

**MATH 7390** Topics in Mathematics - Level 7 (3 semester credit hours) May be repeated for credit as topics vary. Instructor consent required. (3-0) T

**MATH 8V02** Individual Instruction in Mathematics (1-6 semester credit hours) Pass/Fail only. May be repeated for credit as topics vary. Instructor consent required. ([1-6]-0) S

**MATH 8V04** Topics in Mathematics - Level 8 (1-6 semester credit hours) Pass/Fail only. May be repeated for credit as topics vary. Instructor consent required. ([1-6]-0) R

**MATH 8V07** Research (1-9 semester credit hours) Open to students with advanced standing subject to approval of the Graduate Advisor. Pass/Fail only. May be repeated for credit. Instructor consent required. ([1-9]-0) S

**MATH 8V99** Dissertation (1-9 semester credit hours) Pass/Fail only. May be repeated for credit. Instructor consent required. Prerequisite: Open to PhD students only. ([1-9]-0) S